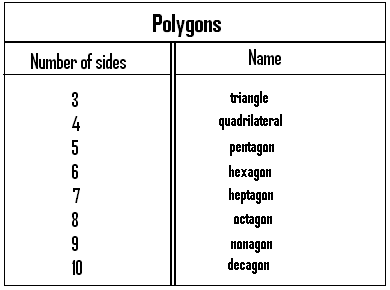
**CHAPTER SEVEN**

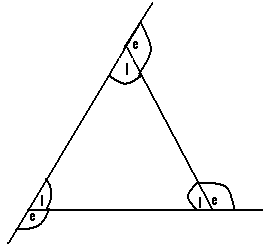
**POLYGONS**

Definition:

A polygon is a plane figure which is bounded by straight lines.



* A polygon has both interior as well as exterior angles.
* The interior angles of a polygon are those angles which lie within the polygon.
* The exterior angles of a polygon lie outside the polygon.



I = interior angle.

e = exterior angle.

N/B: For any polygon, the sum of the exterior angles = 36

Q1. Calculate the value of each exterior angle of a regular decagon.

Soln.

Decagon has 10 sides and as such 10 exterior angles.

But the sum of the exterior angles of any polygon = 36

⇒ 10 exterior angles = 36

∴ 1 exterior angle =

= 3.

⇒ each exterior angle of a decagon = 3.

Q2. Find the exterior angle of a regular pentagon.

Soln.

Pentagon has 5 sides, and as such 5 exterior angles. But the sum of the exterior angles of a polygon = 36

⇒ 5 exterior angles = 360

⇒ 1 exterior angle =

=

∴ Each exterior angle of the regular pentagon =720. For any polygon, the sum of the exterior angle and the exterior angle at any of its vertices = 18.

**Determination of the interior angle of a regular polygon:**

- We must first determine the value of the exterior angle.

- Using the fact that at any vertex, exterior angle + interior angle = 18.

⇒ interior angle = 18 - exterior angle.

Q1. Calculate the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

⇒ 10 exterior angles = .

∴ 1 exterior angle =

=

But at any vertex, exterior angle + interior angle = 18.

⇒ + interior angle = 18.

Interior angle =18.

The interior angle of the decagon = .

Q2. Find the value of each Interior angle of a triangle.

Soln.

A triangle has 3 sides and as such 3 exterior angles.

⇒ 3 exterior angles =

∴ 1 exterior angle =

= 12.

But at any vertex, interior angle + exterior angle = 18

⇒ Interior angle + 12 = 18

∴ Interior angle =

**Determination of the sum or the total interior angles of a polygon:**

For any polygon, the sum of the interior angles = the number of sides of the polygon the value of one interior angle.

Q1. Calculate the sum of the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

⇒ 10 exterior angles =

∴ 1 exterior angle =

= 3

But at any vertex, interior angle + exterior angle = 1

⇒ Interior angle + 360 = 1800

⇒ Interior angle = 180 – 36

⇒ Interior angle = 14.

But the sum of the interior angles of a decagon = interior angle the number of sides.

∴ Sum of interior angles of the decagon = 14

Q2. Find the sum of the interior angles of a regular octagon.

Soln.

Octagon has eight sides and as such eight exterior angles.

⇒ 8 exterior angles =

∴ 1 exterior angle =

= .

But at any vertex, exterior angle + interior angle = 18

∴ + interior angle = 1

⇒ Interior angle = 180 – 45 = .

But the sum of interior angle = the number of sides of the polygon angle = 8 = .

Q3.The interior angles of a regular triangle are marked and Find the actual values of each of these angles.

N/B: First calculate the sum of the interior angles of the triangle.

Soln.

Triangle has 3 exterior angles

⇒ 3 exterior angles =

∴ 1 exterior angle =

= .

But at any vertex, exterior angle + interior angle =

⇒ + interior angle =

⇒ Interior angle = - = .

But the sum of the interior angles of the triangle = the number of sides interior angle = 3 60 = .

But the interior angles of the triangle are given as and The sum of these interior angles = +

= .

But the sum of the interior angles of the polygon or triangle =

⇒

⇒

⇒.

∴ The angle marked 0 + 200 = 400.

The angle marked 100 + 5x0 = 100 + 50(10) = 10 + 500 = 600.

Lastly, the angle marked .

Q4. The angles of a pentagon are marked .

1. Find the value of x.
2. Determine the value of each of those angles.

Soln.

Pentagon has 5 exterior angles.

5 exterior angles =

∴1 exterior angle =

= .

But at any vertex, exterior angle + interior angle =

⇒

⇒ interior angle = 0.

Sum of the interior angles of the pentagon = number of sides interior angle

= .

The given angles which are are the interior angles of the pentagon.

Sum of these interior angles =

= 7x + 50.

Since the sum of the interior angles of the pentagon has been calculated to be equal to

⇒ ∴ x =

The value of the angle marked .

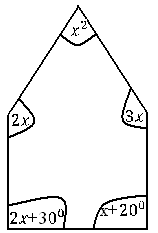
The value of the one marked x + .

The angle marked x + 25 = 70 + 25 = .

The angle marked 2x = 2 .

Lastly, the angle marked 2x + 5 = 2(70) + 5 = 140 + 5 =

Q5.



Determine the value of *x*.

Soln.

The given figure has five sides (a pentagon) and as such has five exterior angles.

5 exterior angles =

⇒ 1 exterior angle =

=

But at any vertex, interior angle + exterior angle

⇒ + interior angle =

⇒ interior angle = 180 – 72 = .

The sum of the interior angles of the pentagon = number of sides .

The sum of the interior angles of the given figure = x + 2x + 3x + 2x +

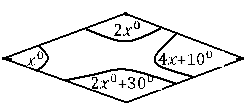
⇒ 9x +

⇒ 9x = 540 – 50 = 490

⇒ 9x = 490 ⇒ x =

∴ x =

Q6.



Calculate the value of *x.*

Soln.

The given figure is a quadrilateral and as much has four exteriior angles.

4 exterior angles =

∴ 1 exterior angle =

= .

But at a vertex, exterior angle + interior angle =

⇒ 9

⇒ interior angle = 180 – 90 = 90.

Sum of the interior angles of a polygon = number of sides interior angle

= 4 .

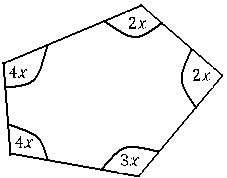
The sum of the interior angles of the given polygon =

⇒

⇒ 9x =

∴ x =

Q7.



Find the value of x.

Soln.

The given figure has five sides (pentagon), and as such has five exterior angles.

5 exterior angles =

∴ 1 exterior angle =

= .

But at a vertex, interior angle + exterior angle =

⇒ interior angle +

∴ interior angle = 18

⇒ Interior angle =

Sum of the interior angles of the given figure = number of sides interior angle

= 5.

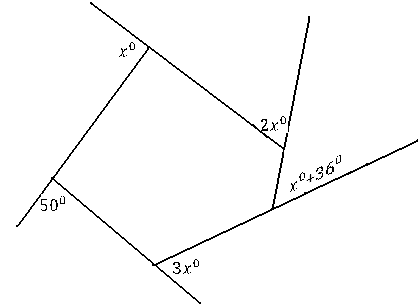
The sum of the interior angles of the given figure = 4x + 4x + 2x + 2x + 3x = 15x

∴ 15x =

⇒ x = .

N/B: The sum of the exterior angles of any polygon is equal to .

Q8.



Find the value of x.

N/B: All the given angles are exterior angles.

Soln.

The given figure is a polygon (pentagon) and has five exterior angles.

But the sum of the exterior angles of a polygon =

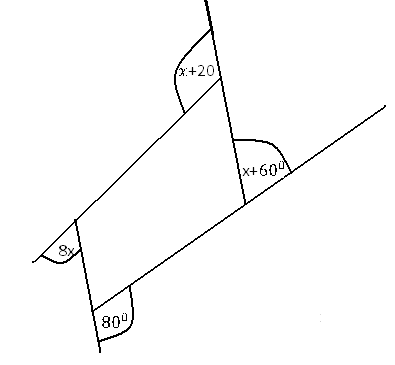
⇒

⇒ 7x + 86 = 360 => 7x = 360 - 86 = 2740.

∴ x =

⇒ x = .

Q9



Determine the value of x.

N/B: The exterior angles of the given figure are +and .

Soln.

Sum of the exterior angles of the given figure = + +

= 10x + 160.

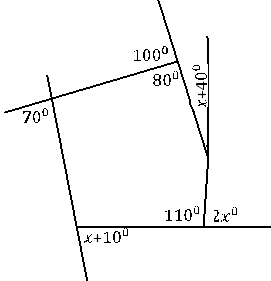
Since the given figure is a polygon (i.e a quadrilateral), then the sum of its exterior angles is

⇒ 10x + 160 =

⇒ 10x = 360 – 160 = 200

⇒ x =

Q10.



Calculate the values of the angles marked and x + 4.

Soln.

The sum of the exterior angles of the given figure which is a polygon

= 70 + 100 + x + 40 + 2x + x + 10 = 220 + 4x.

Since the sum of the exterior angles of a polygon =

⇒ 4x = 360 – 220 = 140

⇒ x = .

The value of the angle marked 2x = 2(.

Also the value of the angle marked x + 40 = 35 + 40 = 75..

**Determination of the number of sides of a polygon:**

The number of sides of any polygon =

Q1. The exterior angle of a polygon is

Soln.

Number of sides = =

Q2.Given that the exterior angle of a polygon is determine its number of sides.

Soln.

Number of sides =

=

Q3. Determine the number of sides of a polygon, whose interior angle is .

N/B: First find the exterior angle and use it to divide

Soln.

At any vertex, exterior angle + interior angle =

⇒ exterior angle +

⇒ exterior angle =

⇒ exterior angle = .

Number of sides =

Q4. Determine the name of a polygon, whose interior angle is

N/B: By determining the number of sides, we can know the name of such a polygon.

Soln.

At any vertex, exterior angle + interior angle = .

∴ exterior angle +

⇒ exterior angle = 180 – 135 = 45.

Number of sides = =

∴ The polygon has 8 sides and as such it is an octagon.